Structural identification for layered composites through a waves and finite elements

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Abstract—**A novel approach for identifying the geometric and material characteristics of layered composite structures through an inverse wave and finite element methodology is proposed. These characteristics are obtained through multi-frequency single shot measurements. It is emphasized that the success of the approach is independent of the employed excitation frequency regime, meaning that both structural dynamics and ultrasound frequency spectra can be employed. Since a full FE description is employed for the periodic composite, the presented approach is able to account for structures of arbitrary complexity. The procedure is applied to a sandwich panel with composite facesheets and results compared well with two wave-based characterization techniques: the Inhomogeneous Wave Correlation method and the Transition Frequency Characterization method.**

Keywords—**Structural identification, Non-Destructive Evalua-tion, Finite Elements, Wave Propagation, Layered Structures, Ultrasound**

I. Introduction

Composite structures are increasingly used in modern aerospace and automobile industries due to their well-known benefits. However, the verification and Non-Destructive Evaluation (NDE) of the actual mechanical properties of the assembled layered structure remains a very much open engineering challenge. Over the past decades, different system identifica-tion methods in the time domain [1], frequency domain [2] and time-frequency domain [3] have been proposed.

One can cite the experimental method for the characterization of Nomex cores [4], or the vibratory identification technique proposed in Matter et al. [5]. Finite Element (FE) based wave methods assume a full 3D displacement field and are therefore capable of capturing the entirety of wave motion types in the waveguide under investigation in a very accu-rate and efficient manner. FE-based wave propagation within periodic structures was firstly considered in [6]. The Wave and Finite Element (WFE) method was introduced in [7], [8] in order to facilitate the post-processing of the eigenproblem solutions. The WFE has recently found applications in predict-ing the vibroacoustic and dynamic performance of composite panels and shells [9], [10], [11] , with complex periodic structures [12], [13] having been investigated. The variability of vibroacoustic transmission through layered structures [14], [15], [16], as well as structural identification [17] have also been considered.

The principal novel contribution of this work is the development of a comprehensive methodology coupling periodic structure theory to FE in order to identify the characteristics of each individual layer of a composite structure. The paper is organised as follows: Sec.II presents the inverse WFE-based computational scheme for identification of layered structures. In Sec. III, numerical and experimental case studies are presented for validating the exhibited identification approach. Conclusions are eventually drawn in Sec. IV.

II. An inverse wave and finite element methodology for structural identification

An arbitrarily complex and periodic in the x direction waveguide is illustrated in Fig.1. The structure may comprise an arbitrary number of layers which may be anisotropic. It is assumed that some of the structural characteristics are unknown (or even variable over time) and need to be evaluated through a non-destructive evaluation process. The identifiable properties include the thickness, density as well as the material characteristics of each individual layers. In the following, a wave and finite element scheme is employed in order to recover the required properties of the layered structure through the acquired propagating wave data.

A. Obtaining the reference and the WFE wave characteristics

The required data to be extracted and later fed into the structural identification process are the wave phase speeds (or wavenumbers) of specific wave types propagating within the laminate under investigation. A number of methods can be employed for exciting and measuring specific propagating wave modes within a composite structure. Piezoelectric [18] or even non-contact laser actuation [19] can be employed for exciting and measuring wave properties in the ultrasound frequency range, while within the structural dynamics spectrum more conventional shaker and accelerometer devices can be employed

An illustration of the configuration is depicted in Fig.2. The waveguide is excited at a specified central harmonic signal of frequency f_0 at a location $x = x_0$ and the signal is monitored at location $x = x_1$, after which the signal has travelled over a distance of $L = x_1 - x_0$. Once the experimental or numerical signal measurements are logged, the wavenumbers and group velocities of the excited waves can be easily determined.

Time histories are initially registered at the excitation and monitoring locations. The maximum amplitudes of the time history signals $x(t)$ are obtained from the Hilbert Transforms $H[x(t)]$ of the acquired signals in the time domain. Hilbert Transform $H[x(t)]$ of the acquired time signal $x(t)$ is used

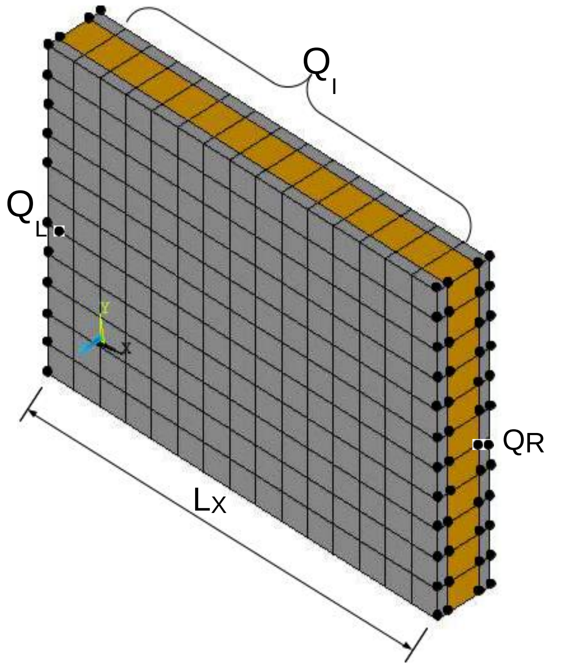

Fig. 1. Caption of the WFE modelled composite waveguide with the left and right side nodes Q_L, Q_R bullet marked. The range of interior nodes Q_I is also illustrated.

to evaluate the main attributes of $x(t)$. The signal envelope is determined at emission, x_0 and arrival, x_1 while the time delay is defined by the time difference between the maximal amplitudes of the envelopes. The total Time of Flight of the wave signal from the point of excitation to the monitoring point is measured as the time difference $t(x_1)-t(x_0)$ between the maximum amplitudes of the excited and the monitored signal envelopes. In ultrasonic NDE, the wavenumber of the wave package (of wave type j) is straightforward to obtain as:

$$\omega$$

H where the phase velocity of the signal c_p can be obtained from its ToF and its propagation distance L. It is noted that the phase velocity for a non-dispersive wave is equal to its group velocity.

H The propagation constants for the elastic waves travelling in the x direction can be sought through the forward Wave and Finite Element (WFE) scheme as described in [17]. The advantage of the WFE approach is the fact that since the excitation frequency is controlled and known, an unlimited number of eigenvalues (for the same wave) can be extracted for the corresponding number of frequencies. Since each resulting j_{th} eigenvalue (propagation constants for each wave type) can be expressed as

$$k_{j,rf} = \frac{}{c_p}$$

$$\gamma_{j,fe} = e^{-ik_{j,fe} L_x}$$

the corresponding wavenumber k_j can be given by

$$k_{j,fe} = \frac{\log \gamma_{j,fe}}{-iL_x}$$

which can be directly compared to the reference wavenumber values $k_{j,rf}$.

B. Formulation of the identification objective function

The objective function of the identification process to be minimized is then obtained through a least squares approach as

$$F(P) = \sum_{H}^{H} (k_{m,rf} - k_{m,fe})^2 \tag{4}$$

with $k_{m,rf}$ and $k_{m,fe}$ being measured and calculated respectively at frequency ω_m for the same wave type, while P is the vector of parameters to be identified; in the very general case this is expressed as

$$P = \left[E_{x,1}, E_{y,1}, E_{z,1}, \nu_{xy,1}, \nu_{xz,1}, \nu_{yz,1}, G_{xy,1}, G_{xz,1}, G_{yz,1}, h_1, \rho_{m,1}, \cdots \rho_{m,l_{MAX}} \right]^T$$

for layers $l \in [1, l_{max}]$. In the above, m_{max} is the total number of reference eigenvalues which can be used in the identification procedure. It is obvious that the minimum required m_{max} is equal to the number of parameters to be identified, however results for additional frequencies will generally improve the precision of the identification process. An excessive m_{max} is undesired, as for each computation of F, an equivalent number of eigenproblems needs to be solved.

In order to accelerate the Newton-like iterative scheme, the first (or even the second) gradient of the objective function $\frac{\partial F}{\partial \beta_i}$ may be provided for each sought structural property β_i as

$$\frac{\partial F}{\partial x} = \sum^{m_{MAX}} 2k_{m,fe} \frac{\partial k_{m,fe}}{} - 2k_{m,rf} \frac{\partial k_{m,fe}}{} \tag{6}$$

The generic iterative procedure of the post-processing identification process is presented in Algorithm 1.

III. NUMERICAL AND EXPERIMENTAL CASE STUDIES

(1)

A. Numerical validation of the identification scheme

The numerical case study relates to identifying the thick-ness, density and Young’s modulus for a monolayer metallic structure under investigation. The properties exhibited under Structure I (see Table 1) are employed. A longitudinal pressure wave excitation is numerically imposed at a cross section of the modelled structure. A general presentation of the measurement process is depicted in Fig.3).

The propagating waveform is depicted in Fig.4 for six wave pulses of different frequencies. As expected, negligible dispersion occurs for all six pulses, thanks to the high number of cycles n_0 employed for the Hanning window process, as well as to the non-dispersive nature of pressure waves.

Six wavenumber measurements are recovered for an equiv-alent number of different ultrasonic frequencies, namely from

(2) 100kHz to 350kHz with a step of 50kHz. The reference wave characteristics related to the recovered wavenumber values are shown in Fig.5. These are retained for comparison with the WFE obtained results for the same propagating wave type

(3) which will form the objective function of the identification

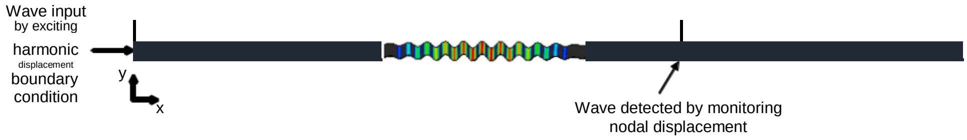

Fig. 2. Illustration of the suggested configuration for obtaining the reference wave characteristics to be later compared with the WFE ones. All simulations are performed using ANSYS V4.15. Three-dimensional solid brick elements are employed for enhanced accuracy and a minimum mesh density of 15 elements per wavelength is retained.

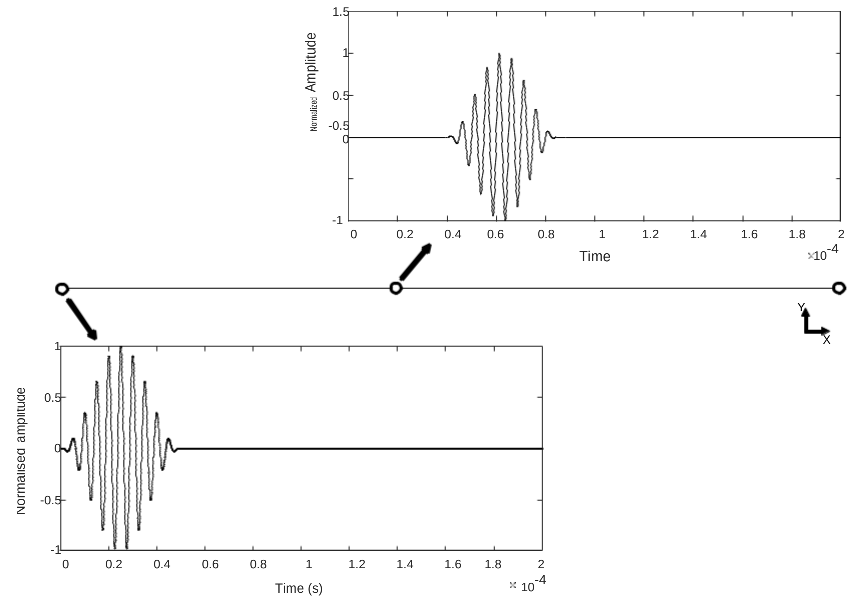

Fig. 3. General representation of the ToF measurements. The pulse input is generated using an excitation device at the input point while the time delay is measured at the monitoring point. Note that better results are obtained when no edge reflections are interfering with the registered pulse.

problem. The same process is repeated for a flexural wave propagating within the monolayer structure with the results also presented in the same figure.

Once the reference wave characteristics $k_{m,rf}$ are estab-lished, the objective function F can be established as a function of the structural properties to be identified E, ρ and h. A single element is employed for the formulation of the WFE model which results in very fast eigenproblem solutions. An identification criterion equal to 10 is employed (suggesting that any local minimum with a value less than that would be considered as a solution). The minimization process was completed in 58 iterations each of which lasted approximately 8 seconds, resulting in a total computation time of 460s on a conventional laptop device. This suggests that employing dedicated optimization software and high-performance com-puting equipment would radically reduce this amount of post-processing. The final value of the objective function when pressure wave measurements were employed was of the order of 10^{-1}. The second best identified solution gave an objective function value at the order of 10^{1}, therefore confirming the optimality of the result. The identified parameters are exhibited in Table 1 and are in excellent agreement with the ones initially used in the full FE model (maximum divergence is considerably less than 1%). The result therefore validates the accuracy and robustness of the proposed scheme.

TABLE I

PROPERTIES OF NUMERICALLY MODELLED STRUCTURAL LAYERS AND IDENTIFIED CHARACTERISTICS THROUGH THE INVERSE WFE SCHEME

Structure I
ρ = 7850 kg/m^3 h = 1 mm E = 170 GPa ν = 0.29
Identified structural characteristics of each layer
ρ = 7857.43 kg/m^3 h = 0.9973 mm E = 174.32 GPa

B. *Experimental validation of the identification scheme*

The proposed identification strategy is applied to a sandwich structure, and results are compared with the ones obtained experimentally in Droz et al. [20] from IWC method, static experiments and the Transition Frequency Characterization (TFC).

The structure is a rectangular (60 cm × 288 cm) sandwich plate whose constitutive materials are a 10 mm-thick Nomex honeycomb core involving a 3.2 mm cell size, while propa-gation is considered in the W-direction. The sandwich's skins

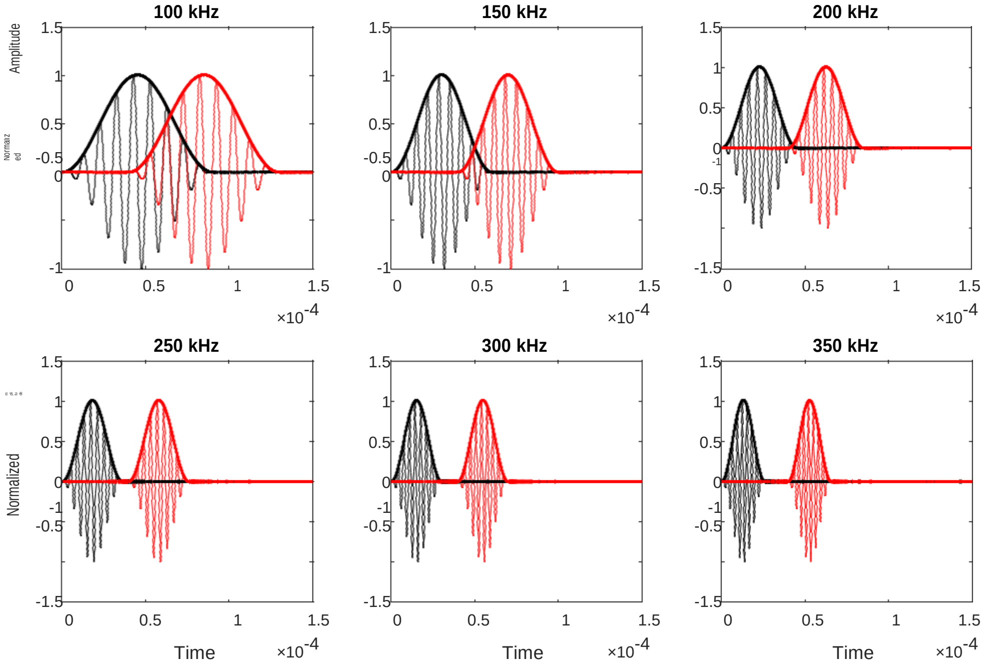

Fig. 4. Time acquisition at $x = 0$ (black curves) and $x = 3$cm (red curves) with the wave envelopes depicted in the monolayer structure. The number of cycles is n_0=9. The ToF is measured at the maximal amplitude of Hilbert transform (solid lines) signal.

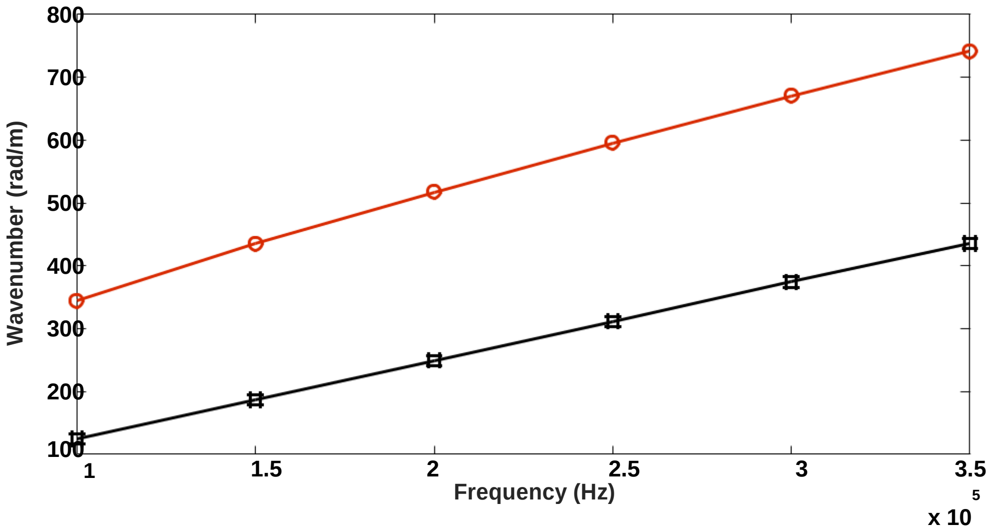

Fig. 5. Reference wavenumber values obtained through a numerical solution of the full FE model for the monolayer structure. Results for a pressure propagating wave ( ) and a flexural propagating one ( ).

are 0.6 mm-thick Hexforce with multi-axial, carbon-reinforced fibres. The density of the skins given by the manufacturer is $\rho_s = 1451$ kg.m^{-3} and the core's density is $\rho_c = 99$ kg.m^{-3}.

Taking into account the material characteristics provided by the manufacturer for the layered panel, the WFE iterative process is formed and the properties of the panel are identified through the presented Newton-like minimization scheme. The process depicted in Fig.6 and detailed by Algorithm 1 was programmed and executed with the experimentally obtained flexural wavenumbers serving as the reference measured values. The structural parameters to be identified are the Young's modulus of the facesheet and the shear modulus of the core in the direction of wave propagation. A new design was therefore generated after each iteration, taking into account the first derivative of F. After converging to a minimum of F, the final value of the objective function was compared to the identification criterion. If the identification condition was not satisfied, a drastically altered design was evaluated by the iterative algorithm. An identification criterion equal to 10 was employed while the minimization process converged in 91 iterations each of which lasted approximately 14 seconds, resulting in a total computation time of 1274s on a conventional

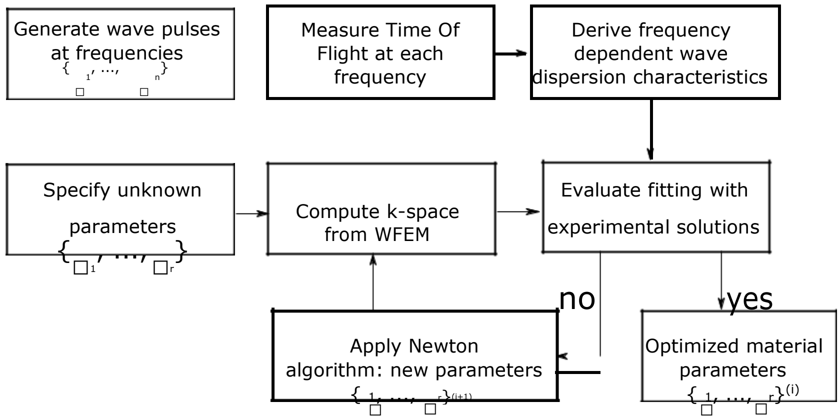

Fig. 6. Experimental procedure for the WFE-based model updating strategy.

Algorithm 1 Newton-like iterative scheme for identifying the parameters of a layered structure

1: Input measured reference wave characteristics. Determine total number of local minima to be investigated and evaluated. Define identification criterion for objective function F
2: $i \leftarrow 1$ Input structural parameters for initial design to be evaluated
3: Substitute new set of structural parameters in symbolic expressions of M, K, $\frac{\partial K}{\partial \beta_i}$, $\frac{\partial M}{\partial \beta_i}$ and formulate the corresponding matrices for the periodic unit cell of the layered design under investigation
4: Solve the WFE eigenproblem for design i. Compute WFE wave velocities and wavenumbers
5: Compute F and the sensitivity values $\frac{\partial F}{\partial \beta_i}$ for each structural parameter β_i to be recovered
6: **if** $dF <$ Solution convergence criterion **then**
7: Solution corresponds to a local minimum
8: **if** $F <$ Identification criterion **then**
9: Solution corresponds to global identification solution and process can end
10: **else**
11: Radically alter the structural parameters and go to Step 3
12: **end if**
13: **else**
14: Use $\frac{\partial F}{\partial \beta_i}$ in order to alter structural parameters for converging towards a local minimum. $i \leftarrow i + 1$ (next solution step). Go to Step 3
15: **end if**

laptop device.

The identified Young's modulus for the skins of the laminate and the shear modulus of the honeycomb core through the presented scheme are compared to those of the experimental methods as:

$$E_{manuf} = 70 \text{ GPa} \quad \text{and} \quad G_{manuf} \in [30 - 38] \text{ MPa}$$
$$E_{IWC} = 62 \text{ GPa} \quad \text{and} \quad G_{IWC} = 37.8 \text{ MPa}$$
$$E_{TFC} = 69.8 \text{ GPa} \quad \text{and} \quad G_{TFC} = 36.5 \text{ MPa}$$
$$E_{WFE} = 69.5 \text{ GPa} \quad \text{and} \quad G_{WFE} = 37.1 \text{ MPa}$$

which are both in very good agreement with the values provided by the experimental methods, therefore experimentally validating the exhibited computational scheme.

IV. Concluding Remarks

In this work a new identification technique, based on FE modelling and the properties of propagating waves in layered structures, is developed and applied. The principal contribution resulting from this work is a robust numerical NDE procedure for recovering effective structural parameters of complex, layered composites.

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